

THE DOMAIN AND RANGE OF A FUNCTION
ALGEBRA 2 WITH TRIGONOMETRY

Because functions convert values of inputs into value of outputs, it is natural to talk about the sets that represent these inputs and outputs. The **set of inputs** that result in an output is called **the domain** of the function. The **set of outputs** is called **the range**.

Exercise #1: Consider the function that has as its inputs the months of the year and as its outputs the number of days in each month. In this case, the number of days is a function of the month of the year. Assume this function is restricted to non-leap years.

(a) Write, in roster form, the set that represents this function's domain.

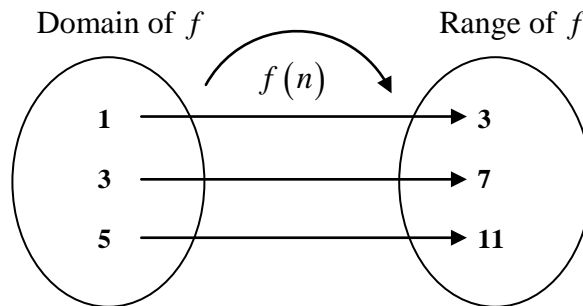
(b) Write, in roster for, the set that represents this function's range.

{January, February, March, ..., December}

{28, 30, 31}

Exercise #2: State the range of the function $f(n) = 2n + 1$ if its domain is the set $\{1, 3, 5\}$. Show the domain and range in the mapping diagram below.

Range: {3, 7, 11}



Exercise #3: The function $y = g(x)$ is completely defined by the graph shown below. Answer the following questions based on this graph.

(a) Determine the minimum and maximum x -values represented on this graph.

$x_{\min} = -3$ and $x_{\max} = 6$

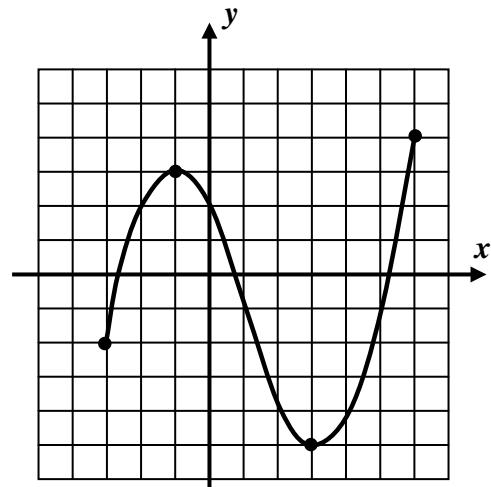
(b) Determine the minimum and maximum y -values represented on this graph.

$y_{\min} = -5$ and $y_{\max} = 3$

(c) State the domain and range of this function using set builder notation.

Domain: $\{x \mid -3 \leq x \leq 6\}$

Range: $\{y \mid -5 \leq y \leq 3\}$



Some functions, defined with graphs or equations, have domains and ranges that stretch out to infinity. Consider the following exercise in which a standard parabola is graphed.

Exercise #4: The function $f(x) = x^2 - 2x - 1$ is graphed on the grid below. Which of the following represent its domain and range written in interval notation?

(1) Domain: $[-2, 4]$

Range: $[-4, 6]$

(3) Domain: $(-\infty, \infty)$

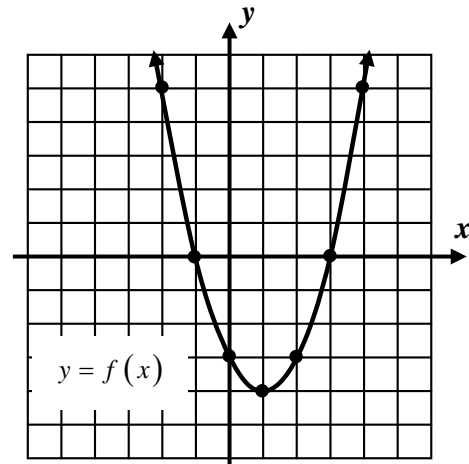
Range: $[-4, \infty)$

(2) Domain: $[-2, 4]$

Range: $(-4, \infty)$

(4) Domain: $(-2, 4)$

Range: $(-4, 6)$



For most functions defined by an algebraic formula, the domain consists of the set of all real numbers, given the concise symbol \mathbb{R} . Sometimes, though, there are restrictions placed on the domain of a function by the structure of its formula. Two basic restrictions will be illustrated in the next few exercises.

Exercise #5: The function $f(x) = \frac{2x+1}{x-4}$ has outputs given by the following calculator table.

(a) Evaluate $f(1)$ and $f(6)$ from the table.

$f(1) = -1$ and $f(6) = 6.5$

(b) Why does the calculator give an ERROR at $x = 4$?

It gives an error because when $x = 4$ we are forced to divide by zero.

X	Y1
1	-1
2	-2.5
3	-3
4	ERROR
5	11
6	6.5
7	5

X=7

(c) Are there any values except $x = 4$ that are not in the domain of f ? Explain.

No. There are no other values of x which when substituted into the function as inputs would force the function to divide by zero. Thus, all other values of x are in the domain of this function.

Exercise #6: Which of the following values of x would not be in the domain of the function $y = \sqrt{x+4}$? Explain your answer.

(1) $x = 0$

(3) $x = -3$

(2) $x = 5$

(4) $x = -8$

$x = -8$ is not in the domain of this function because upon substitution it would force the function to take the square root of a negative number. In the Real Number System, this is not possible.

(4)



**THE DOMAIN AND RANGE OF A FUNCTION
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

SKILLS

1. A function is given by the set of ordered pairs $\{(2, 5), (4, 9), (6, 13), (8, 17)\}$. Write its domain and range in roster form.

Domain: $\{2, 4, 6, 8\}$

Range: $\{5, 9, 13, 17\}$

2. The function $h(x) = x^2 + 5$ maps the domain given by the set $\{-2, -1, 0, 1, 2\}$. Which of the following sets represents the range of $h(x)$?

(1) $\{0, 6, 10, 12\}$

(3) $\{5, 6, 9\}$

(2) $\{5, 6, 7\}$

(4) $\{1, 4, 5, 6, 9\}$

$f(-2) = 9$

$f(-1) = 6$

$f(0) = 5$

$f(1) = 6$

$f(2) = 9$

(3)

3. Which of the following values of x would *not* be in the domain of the function defined by $f(x) = \frac{x-2}{x+3}$?

(1) $x = -3$

(3) $x = 3$

(2) $x = 2$

(4) $x = -2$

$x + 3 \neq 0$

$x \neq -3$

(1)

4. Determine any values of x that do not lie in the domain of the function $f(x) = \frac{3x+2}{2x-10}$. Justify your response.

$2x - 10 = 0$
 $2x = 10$
 $x = 5$

The only value of x not in the domain of the function is $x = 5$ because it is the only value that forces the function to divide by zero.

5. Which of the following values of x *does* lie in the domain of the function defined by $g(x) = \sqrt{2x-7}$?

(1) $x = 0$

(3) $x = 3$

(2) $x = 2$

(4) $x = 5$

$g(5) = \sqrt{2(5)-7} = \sqrt{10-7} = \sqrt{3}$

All other values force the function to take the square root of a negative number.

(4)

6. Which of the following would represent the domain of the function $y = \sqrt{6-2x}$?

(1) $\{x : x > 3\}$

(3) $\{x : x \leq 3\}$

(2) $\{x : x < 3\}$

(4) $\{x : x \geq 3\}$

$6 - 2x \geq 0 \Rightarrow -2x \geq -6$

$\frac{-2x}{-2} \geq \frac{-6}{-2} \Rightarrow x \leq 3$

(3)



7. The function $y = f(x)$ is completely defined by the graph shown below.

(a) Evaluate $f(-4)$, $f(3)$, and $f(6)$.

Read y-coordinates for the given x:

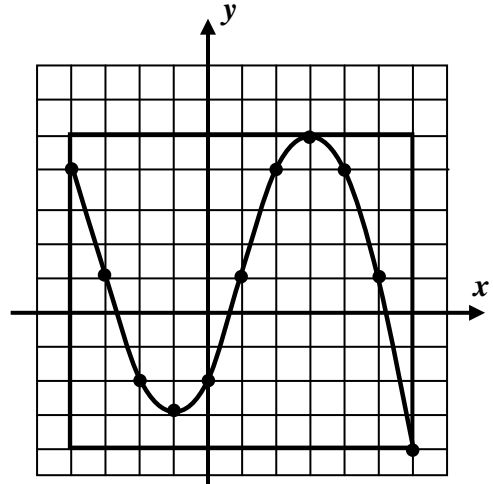
$$f(-4) = 4, f(3) = 5, \text{ and } f(6) = -4$$

(b) Draw a rectangle that circumscribes (just surrounds) the graph.

(c) State the domain and range of this function using interval notation.

Domain: $[-4, 6]$

Range: $[-4, 5]$



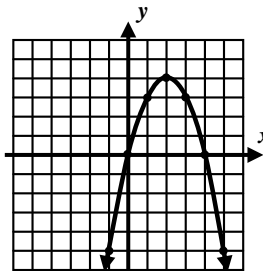
8. Which of the following represents the range of the quadratic function shown in the graph below?

(1) $(4, \infty)$

(3) $(-\infty, 4)$

(2) $(-\infty, 4]$

(4) $[4, \infty)$



(2)

APPLICATIONS

9. A child starts a piggy bank with \$2. Each day, the child receives 25 cents at the end of the day and puts it in the bank. If A represents the amount of money and d stands for the number of days then $A(d) = 2 + 0.25d$ gives the amount of money in the bank as a function of days (think about this formula).

(a) Evaluate $A(1)$, $A(7)$, and $A(30)$.

$$A(1) = 2.25, A(7) = 3.75, \text{ and } A(30) = 9.50$$

(c) Explain why the domain does not contain the value $d = 2.5$.

The domain only contains whole numbers because the child is only receiving the 25 cents at the end of each day. Thus, no partial day value is in the domain of this function.

(b) For what value of d will $A(d) = \$10.50$.

$$\begin{aligned} 2 + 0.25d &= 10.50 \\ 0.25d &= 8.50 \Rightarrow d = 34 \end{aligned}$$

(d) Explain why the range does not include the value $A = \$3.10$.

The outputs to this function are multiples of 0.25 starting with 2.00. For example, 2.00, 2.25, 2.50, 2.75, etc. Thus, an output value of 3.10 will never be achieved by any input in the domain.

