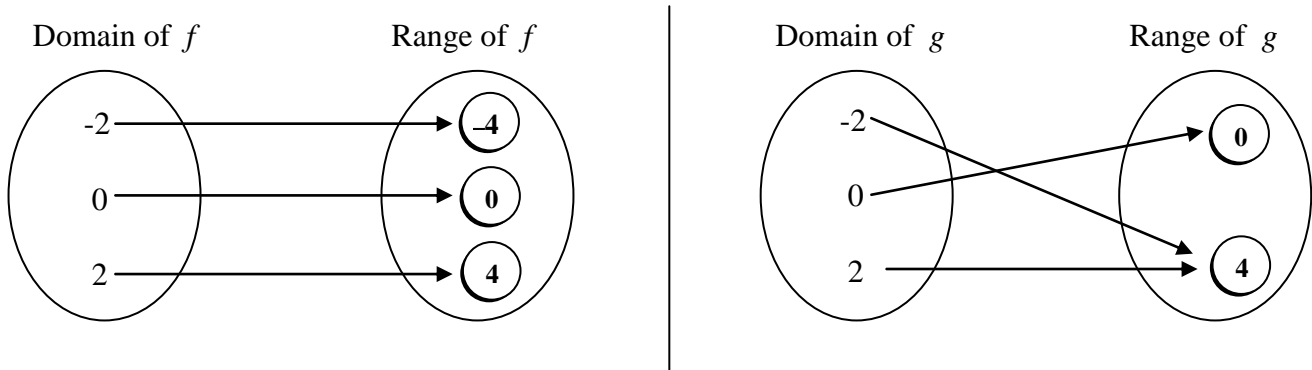


ONE-TO-ONE FUNCTIONS ALGEBRA 2 WITH TRIGONOMETRY

Functions as rules can be divided into various categories based on shared characteristics. One category is comprised of functions known as one-to-one. The following exercise will illustrate the difference between a function that is one-to-one and one that is not.

Exercise #1: Consider the two simple functions given by the equations $f(x) = 2x$ and $g(x) = x^2$.

(a) Map the domain $\{-2, 0, 2\}$ using each function. Fill in the range and show the mapping arrows.



(b) What is fundamentally different between these two functions in terms of how the elements of this domain get mapped to the elements of the range.

In the first function, f , each member of the domain gets a unique member of the range (its own), whereas in the second function, g , members of the range are repeated.

ONE-TO-ONE FUNCTIONS

A function $f(x)$ is called one-to-one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$.
(In other words, different inputs give different outputs.)

Exercise #2: Of the four tables below, one represents a relationship where y is a one-to-one function of x . Determine which it is and explain why the others are not.

(1)

x	y
4	2
4	-2
9	3
9	-3

Not even a function due to some inputs having more than one output.

(2)

x	y
-2	1
-1	0
0	1
1	2

Although this is a function, it is not one-to-one because each input does not receive a unique output.

(3)

x	y
1	2
2	4
3	8
4	16

This is a one-to-one function because no inputs or outputs are repeated.

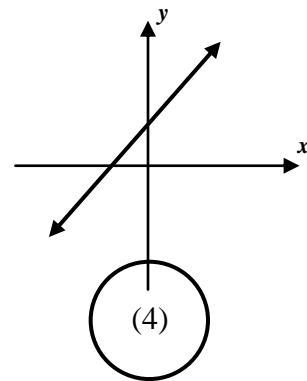
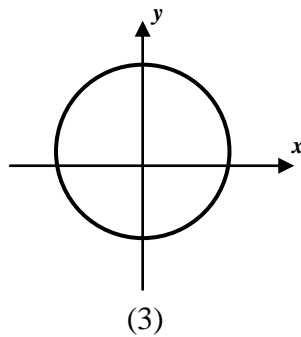
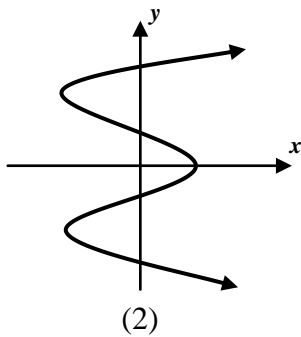
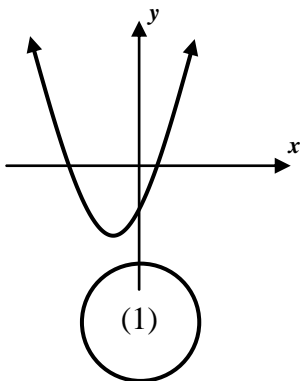
(4)

x	y
-3	10
-2	9
-1	7
-2	10

This is not even a function because inputs are repeated.



Exercise #3: Consider the following four graphs which show a relationship between the variables y and x .



(a) Circle the two graphs above that are functions. Explain how you know they are functions.

(1) and (4) are both functions. This is easy to determine using the Vertical Line Test.

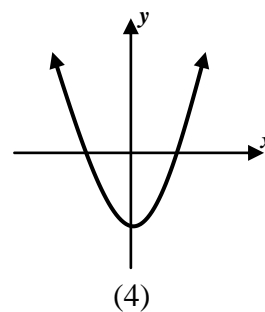
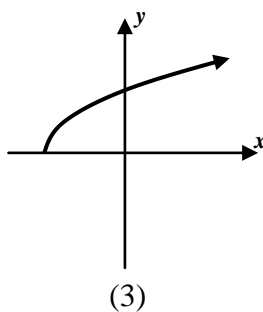
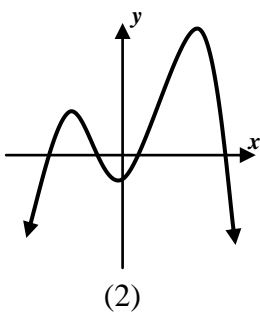
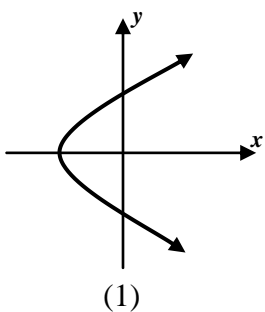
(b) Of the two graphs you circle, which is one-to-one? Explain how you can tell from its graph.

(4) is a one-to-one function because no y -values have occurred more than once.

THE HORIZONTAL LINE TEST

If any given horizontal line passes through the graph of a function at most one time, then that function is one-to-one. This test works because horizontal lines represent constant y -values, hence if a horizontal line intersects a graph more than once, an output has been repeated.

Exercise #4: Which of the following represents the graph of a one-to-one function?



(3)

Exercise #5: The distance that a number, x , lies from the number 5 on a one-dimensional number line is given by the function $D(x) = |x - 5|$. Show by example that $D(x)$ is *not* a one-to-one function.

There are many good examples to show that this is not a one-to-one function. The key here is to find two inputs that have the same output. Consider the following:

$$D(8) = |8 - 5| = |3| = 3$$

$$D(2) = |2 - 5| = |-3| = 3$$

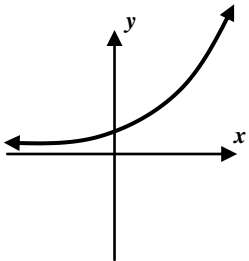
Since $D(8) = D(2)$ an output has been repeated and thus this is not a one-to-one function.



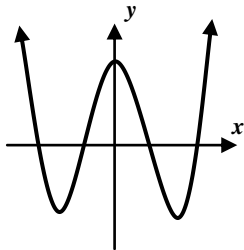
ONE-TO-ONE FUNCTIONS
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

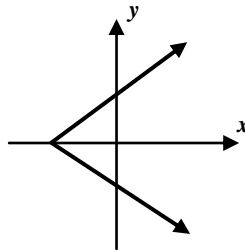
1. Which of the following graphs illustrates a one-to-one relationship?



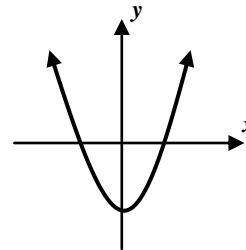
(1)



(2)



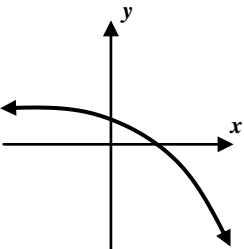
(3)



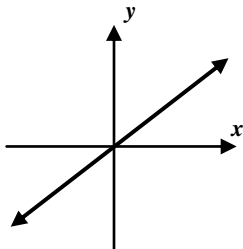
(4)

(1)

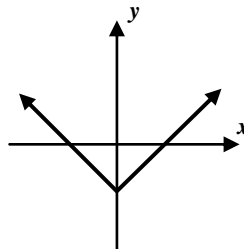
2. Which of the following graphs does *not* represent that of a one-to-one function?



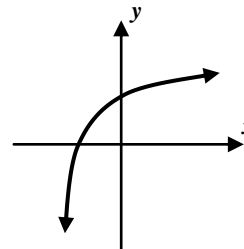
(1)



(2)



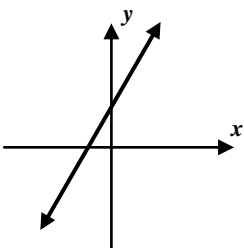
(3)



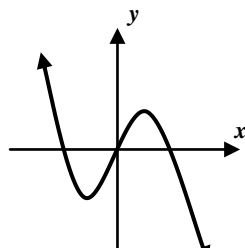
(4)

(3)

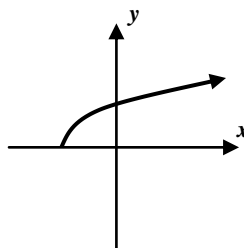
3. In which of the following graphs is each input *not* paired with a *unique* output?



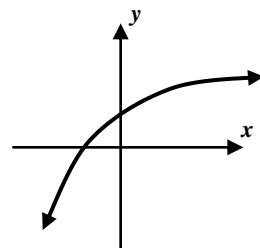
(1)



(2)



(3)



(4)

(2)

4. In which of the following formulas is the variable y a one-to-one function of the variable x ? (Hint – try generating some values either in your head or using TABLES on your calculator.)

(1) $y = x^2$

(3) $y = 2x$

(2) $y = |x|$

(4) $y = 5$

(3)



5. Which of the following tables illustrates a relationship in which y is a one-to-one function of x ?

(1)

x	y
-2	-1
0	-3
2	-1
4	1
6	3

(2)

x	y
-2	-8
-1	-1
0	0
1	1
2	8

(3)

x	y
-2	-5
-1	-4
0	-1
-1	7
-2	5

(4)

x	y
-2	11
-1	-4
0	-5
1	-4
2	11

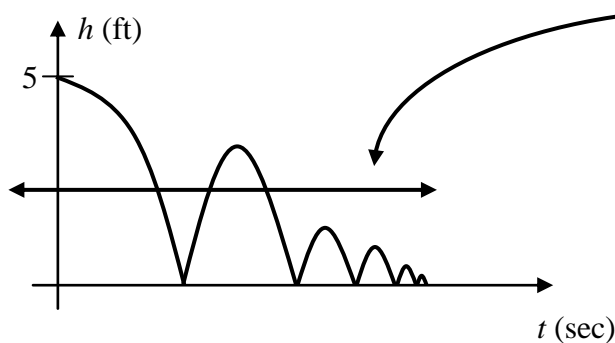
(2)

APPLICATIONS

6. Physics students drop a basketball from 5 feet above the ground and its height is measured each tenth of a second until it stops bouncing. The height of the basketball, h , is clearly a function of the time, t , since it was dropped.

(a) Sketch the general graph of what you believe this function would look like.

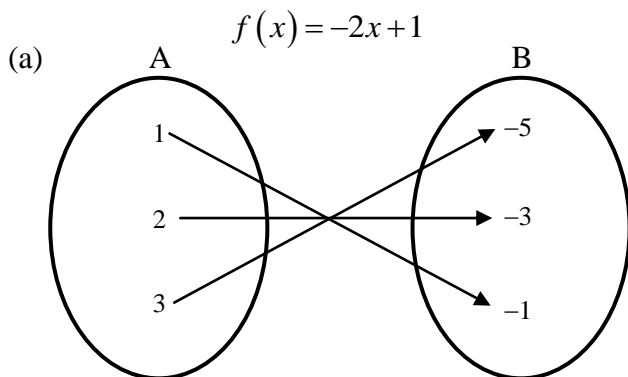
(b) Is the height of the ball a one-to-one function of time? Explain your answer.



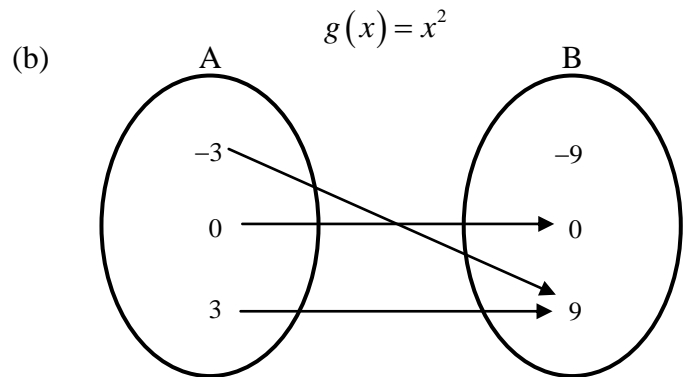
No. Many heights, or outputs, will be repeated as the ball bounces up and down. Notice how it fails the horizontal line test.

REASONING – ONTO FUNCTIONS (OR MAPPINGS) – Another important type of function is known as **onto**. An **onto function or onto mapping** occurs when a function maps the elements from set A to set B and all elements in set B get mapped to. Every member of the output set must be hit for a function to be **onto**.

7. In each case below, show how elements in set A get mapped to elements in set B. Then, state which mapping is **onto** and which is not **onto**.



This mapping is onto because each element in set B was mapped to (had an element of set A as an input).



This mapping is not onto because each element in set B was not mapped to (-9 was missed).

